A Value-per-Joule National Accounting System

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Abstract

Gross domestic product and energy accounts answer different questions. GDP records production under an internationally agreed boundary. Physical energy accounts trace natural inputs, energy products, conversion, losses, and final uses. Measures of quality-adjusted machine service, adoption, realized outcomes, and proof-of-work settlement assurance add still other dimensions. Adding these quantities into one enlarged monetary total would usually double count recorded production or combine stocks, flows, probabilities, and physical services without a common unit. This paper proposes a vector satellite account that keeps those objects linked but separate. The monetary core follows the production identity

$$\mathrm{GDP} = \sum_j \mathrm{GVA}_j + \text{taxes on products} - \text{subsidies on products}.$$

A physical supply-use table balances each energy product. A stage bridge connects primary supply, conversion output, losses, and final use without adding repeated stages. Separate panels distinguish current operational energy from uniquely identified embodied increments. Outcome tables balance produced, adopted, and realized quantities for typed indicators. Artificial-intelligence outcomes appear as physical or service memorandum measures, not as an addition to AI-related output already recorded in value added. Proof-of-work assurance is reported for a named scenario; gross protected positions are exposures, not new production. A constructive result proves that summing primary supply, conversion output, and final energy repeats physical flows. A second result shows why AI value already in gross output and protected asset stocks cannot be added to GDP. Composite indices are optional declared views. Their normalization, weights, and rank sensitivity must be published. A standard-library Python artifact supplies exact-decimal physical tables, bridges, production reconciliation, outcome chains, assurance memoranda, energy-normalized vectors, and weight-sensitivity reports. The account supplements GDP. It does not redefine the SNA production boundary, create a natural country ranking, or claim that joules measure social value.

Keywords: national accounts; SNA; SEEA; energy accounts; satellite accounts; artificial intelligence; proof of work; composite indicators; double counting.

Evidence status. Accounting identities and constructive non-overlap results are PROVED. Synthetic table calculations and software tests are COMPUTATIONAL. Empirical compilation and outcome attribution are CONDITIONAL. No national estimate or country ranking is presented.

Contents

1 Purpose and boundary

National accounts already contain energy industries, data centers, software producers, mining businesses, payment services, equipment manufacture, and the income generated by those activities. A new account should not pretend that these activities are absent from GDP. What is often absent is visibility. Standard classifications can hide cross-cutting activities inside broader industries. Monetary totals do not show how primary energy becomes final energy, how much operational electricity supports a machine service, how often an AI system is adopted, or what threat model underlies a settlement-assurance claim.

The proposed account supplies that visibility without changing the headline production measure. It has four coordinated panels:

1. an SNA production reconciliation with explicit gross value added;
2. an SEEA-style physical energy supply-use table and stage bridge;
3. typed AI, adoption, realized-outcome, and proof-of-work assurance memorandum tables; and
4. optional energy-normalized vectors and declared weight-sensitivity reports.

The panels share industries, products, reporting periods, residence rules, and classification bridges. They do not share one numerator. The architecture is vector-valued because useful machine service, settlement risk, value added, energy, and avoided loss are not naturally commensurate.

1.1 Research question

The paper asks:

> How can a national statistical office publish energy-normalized information on industrial, computational, and settlement outcomes while preserving the production boundary and preventing physical or monetary double counting?

The answer is a reconciled satellite account, not a replacement GDP. The monetary account is copied or disaggregated from the integrated national accounts. Physical energy flows are balanced product by product. Outcome indicators are displayed as memoranda with their own units, system boundaries, counterfactuals, and uncertainty. Any composite uses published normative weights and is accompanied by sensitivity results.

1.2 Contributions

The paper contributes:

1. a precise statement of the current SNA and SEEA standards relevant on 23 July 2026;
2. a table architecture compatible with SNA thematic accounting and SEEA physical energy flows;
3. a proof that adding primary, conversion, and final energy stages repeats flows;
4. controls for operational and embodied-energy scope overlap;

5. balanced outcome chains for AI production, adoption, and realization;
6. a scenario-indexed proof-of-work memorandum that distinguishes protected exposure from modeled avoided loss;
7. a GDP reconciliation that excludes AI value already present in GVA and excludes gross protected assets;
8. a declared composite-index and weight-sensitivity protocol; and
9. an executable synthetic pilot with exact arithmetic and generic tests.

1.3 Nonclaims

The proposal does not:

- redefine GDP or the SNA production boundary;
- measure welfare by energy use;
- claim that energy productivity is new;
- add AI-enabled output on top of output already recorded;
- count gross protected asset value as settlement output;
- add primary energy and converted energy as independent inputs;
- call current operational electricity a lifecycle total;
- turn task counts, probabilities, dollars, and joules into one natural unit;
- infer a causal productivity effect from an observational ratio; or
- produce a universal country league table.

2 Current international statistical standards

2.1 The 2025 System of National Accounts

The edition status has changed recently and needs exact wording. The 2025 System of National Accounts is the current international statistical standard for national accounts. The United Nations Statistical Commission adopted it at its fifty-sixth session in March 2025, and the Economic and Social Council adopted the corresponding resolution on 10 June 2025 [2, 1]. The 2025 SNA updates the 2008 SNA while retaining its basic theoretical framework [3].

As of 23 July 2026, the United Nations Statistics Division provides a searchable pre-edit version. That is the available text cited here. Calling it a "draft SNA" would understate its institutional status because the standard has been adopted. Calling the distributed PDF the final edited publication would overstate its publication status. The precise description is: *the adopted 2025 SNA, consulted in its current pre-edit version* [1, 3].

Implementation is necessarily transitional. The SNA is a conceptual framework, not a schedule requiring every country to switch all releases on one date. The United Nations and partner organizations are developing compilation guidance and technical assistance under an endorsed implementation strategy [4]. A national pilot should state whether its source accounts still follow the 2008 SNA, have migrated to the 2025 SNA, or use a bridge between them [5].

2.2 What the 2025 SNA changes for this paper

Three features are directly relevant. First, the 2025 SNA gives thematic and extended accounts a clear role. Thematic accounts disaggregate and rearrange items already in the integrated framework. Extended accounts may change production or asset boundaries and must label the alternative concepts explicitly [3]. Second, chapter 22 encourages digital supply and use tables and separate "of which" reporting for AI software within computer software. Third, chapter 38 recommends supply-use tables as the organizing framework for a thematic account and allows physical indicators to complement monetary production measures.

This paper chooses a conservative form. The monetary panel is thematic: it disaggregates existing SNA production without extending the headline boundary. Physical and outcome panels are complementary memorandum information. If a compiler later imputes values outside the SNA production boundary, those values belong in a separately labeled extended account and do not silently enter GDP.

2.3 SEEA Central Framework

The SEEA 2012 Central Framework remains the current international statistical standard for environmental-economic accounting [6, 7]. It aligns environmental information with SNA concepts, structures, and accounting rules. It covers physical flows between the environment and economy, flows within the economy, environmental assets, and environmental activities.

An update is underway. The Statistical Commission endorsed an update process in 2024, and global consultations on issues including physical supply-use tables and losses were active in 2026 [8]. The existence of an update does not make the 2012 Central Framework obsolete. The paper uses the adopted 2012 framework and flags places where a future revision may alter terminology or detailed treatment.

2.4 SEEA-Energy

The United Nations published SEEA-Energy in 2019 as the energy subsystem consistent with the Central Framework [9]. It organizes energy-related information in physical and monetary flow accounts and asset accounts. Physical flows are ultimately expressed in joules so energy carriers can be presented in a common unit.

SEEA-Energy requires the physical supply and use identity for each product. Domestic output plus imports equals intermediate and household use, inventory changes, and exports. It also explains why energy flow accounts and energy balances support different views. A gross supply-use presentation can contain both a primary energy product and the electricity made from it. An energy balance follows the conversion chain so analysts do not interpret those repeated stages as independent energy inputs [9, 10].

2.5 Composite-indicator guidance

The OECD and European Commission Joint Research Centre handbook treats normalization, weighting, aggregation, and uncertainty or sensitivity analysis as separate construction steps [11]. It states that weights are value judgments even when statistical methods help select them. Equal weighting is still a weighting choice. It also recommends sensitivity analysis because indicator selection, normalization, weights, and aggregation methods can change scores and rankings.

The optional scalar view in this paper follows that discipline. The vector tables are primary. A composite is a declared analytic lens with published weights, not an official discovery of one natural national objective.

3 Relationship to prior accounting work

3.1 GDP, welfare, and dashboards

GDP measures production, not all welfare. The distinction is established in the national-accounting and beyond-GDP literature [13, 14]. The 2025 SNA now gives well-being and sustainability greater prominence while preserving the integrated production framework [3]. Dashboards and satellite accounts can complement GDP without treating every useful outcome as new production.

The proposed account takes that route. It does not subtract energy from GDP to create "true output." It does not add imputed intelligence or trust. It publishes production, energy, and typed outcomes side by side, with bridges that allow analysis without pretending that the columns share a unit.

3.2 Energy productivity and useful work

Energy intensity and its reciprocal, energy productivity, are long-standing indicators. Patterson distinguishes thermodynamic, physical, economic-thermodynamic, and economic measures [15]. Useful-work research traces conversion from primary energy to end-use services and relates useful work to economic production [16, 17]. Those contributions motivate careful physical stages.

The novelty claim here is narrower. It is the design of one accounting interface for existing production measures, non-overlapping energy stages, AI outcome chains, settlement-assurance memoranda, and declared weighting. The account does not claim to invent energy intensity or useful work.

3.3 Physical flow and input-output methods

Physical supply-use tables, energy balances, and environmentally extended input-output analysis provide mature tools for tracing resources and embodied burdens [9, 10, 18]. Multi-region input-output methods can allocate upstream energy or emissions through global supply chains [19]. Lifecycle assessment likewise requires a goal, functional unit, system boundary, inventory, and allocation rules [20, 21].

These methods do not license adding an embodied allocation to an economy-wide operational total without reconciliation. The same upstream electricity can appear as operational use in the producing sector and as embodied energy allocated to a downstream asset. Both views are useful. Their sum is not a new territorial energy total unless the underlying flow is removed from one panel.

3.4 Digital and AI measurement

The 2025 SNA recognizes AI systems as a special type of software and encourages separate reporting as an "of which" item [3]. It also encourages digital supply and use tables. OECD guidance on digital supply-use tables provides additional compilation structure for digitally ordered and digitally delivered products [12].

Monetary AI production and investment can therefore be present in national accounts. The unresolved problem is not to invent an AI value add-on. It is to connect monetary entries with service quantity, quality, adoption, realized outcome, and energy under explicit measurement boundaries. Papers 3 and 5 in this research program develop those micro-level measurement chains [23, 24]. This paper supplies the national aggregation interface.

3.5 Proof-of-work settlement

Proof-of-work activity can generate recorded production, compensation, asset transactions, and energy use under relevant national-account treatments. The 2025 SNA also gives guidance for crypto assets without corresponding liabilities [3]. None of that implies that gross asset positions are new value added.

Settlement assurance is scenario-conditioned. It depends on confirmation depth, adversarial share, hardware access, costs, incentives, propagation, and time window. Aggregate energy is not sufficient for security [22, 25]. The national account therefore carries a proof-of-work memorandum with the scenario identifier, probability model, protected exposure, expected loss, and energy. It does not publish "protected dollars" as GDP.

4 Design principles

4.1 Keep the integrated account intact

The production-side GDP identity is

$$\text{GDP} = \sum_j \text{GVA}_j + T_p - S_p, \qquad \text{GVA}_j = Y_j - IC_j,$$

where Y_j is sector j's gross output, IC_j is intermediate consumption, and $T_p - S_p$ is taxes less subsidies on products not already included in output valuation. This identity is copied from the integrated account. The satellite does not insert a new line called AI value or trust value.

4.2 Use common classifications and bridge tables

The panels share:

- reporting period and geographic scope;
- SNA residence rules;
- industry and product classifications;
- currency, price basis, and volume method for monetary entries;
- energy carrier and flow classifications for physical entries;
- organization or establishment identifiers where permitted; and
- explicit bridges when a cross-cutting AI or settlement activity spans several standard industries.

A bridge maps source classifications to thematic categories and publishes allocation residuals. It does not overwrite source data.

4.3 Separate stocks, flows, rates, and probabilities

GDP and value added are period flows. An asset position is a stock at a point in time. Energy is a physical flow over an interval. An adoption rate is a ratio. Reorganization risk is a probability conditioned on a scenario. Adding these quantities is invalid before any question of estimation arises.

The account stores each quantity with its type and unit. A monetary label alone does not make two entries additive. Gross protected positions and modeled avoided loss may both be denominated in dollars, but one is an exposure stock and the other is a scenario-conditioned expected flow or change in loss.

4.4 Treat outcomes as memorandum information

Outcome tables balance a causal or operational chain:

$$Q_k^{\text{produced}} = Q_k^{\text{adopted}} + Q_k^{\text{not adopted}},$$

$$Q_k^{\text{adopted}} = Q_k^{\text{realized}} + Q_k^{\text{adopted, not realized}}.$$

These are arithmetic reconciliations, not causal identification. "Realized" requires an indicator-specific definition and evidence rule. For a predictive system it might mean a verified decision meeting a quality threshold. For an automation system it might mean a completed task whose downstream process accepted the result. For a social outcome, randomized or quasi-experimental evidence may be required.

4.5 Publish vectors before composites

For outcome vector $x = (x_1, \ldots, x_K)$ and declared energy $E > 0$, the account may publish

$$v = \left(\frac{x_1}{E}, \ldots, \frac{x_K}{E}\right).$$

The units remain component-specific. Quality-adjusted tasks per joule and modeled avoided dollars per joule are two entries, not one total.

If a user requests a scalar, normalized indicators z_k and weights w_k can form

$$I_w = \sum_{k=1}^{K} w_k z_k, \qquad w_k \geq 0, \qquad \sum_k w_k = 1.$$

The account publishes w, normalization bounds, orientation, aggregation rule, and sensitivity. It does not call I_w natural value.

5 Physical energy supply and use

5.1 Product balance

For energy product r, a physical supply-use table satisfies

$$\sum_a S_{ra} = \sum_b U_{rb},$$

where suppliers a may include domestic producing industries and imports, and users b may include industries, households, inventories, and exports. The identity is product-specific. Balancing all energy products only in the aggregate can hide a fuel discrepancy offset by an electricity discrepancy.

Table 1 is a small synthetic pilot. "Domestic extraction industry" supplies the primary-fuel product. A full SEEA table would also show the preceding natural-input flow from the environment.

Product	Supply or use actor	Joules
Primary fuel	domestic extraction industry, supply	1,000
Primary fuel	energy industries, intermediate use	800
Primary fuel	direct final users, use	200
Electricity	energy industries, supply	300
Electricity	manufacturing, use	120
Electricity	AI services, use	100
Electricity	proof-of-work settlement, use	50
Electricity	households, use	30

Table 1: Synthetic physical energy product supply-use entries. Primary fuel balances at 1,000 J. Electricity balances at 300 J.

The table is gross by product. It correctly records both primary fuel and the electricity produced by transforming part of that fuel. The presence of both products is not itself double counting. The error occurs when an analyst adds their totals and labels the sum as independent energy input. The stage bridge prevents that interpretation.

5.2 Sector linkage

Rows identify energy products; columns or actor fields identify supply and use. The AI and proof-of-work columns are thematic breakouts of industries already present in the source account. They require concordances to standard industry and product classifications. If one establishment performs both AI services and other computing, the compiler must publish the allocation basis, such as metered electricity, machine-hours, or cost shares.

The residual remains visible. Allocating every joule with no residual can create false precision when only facility-level meters exist. A first pilot may publish "unallocated computing electricity" and reduce it as source systems improve.

6 The energy-stage bridge

Let:

- P be primary energy-product supply;
- C be primary energy sent to conversion;
- F_p be direct final primary use;
- X_p be other primary dispositions such as exports or inventory;
- O be converted energy output;
- L be conversion loss;
- F_c be final use of converted energy; and
- X_c be other converted-energy dispositions.

The bridge identities are

$$P = C + F_p + X_p, \qquad C = O + L, \qquad O = F_c + X_c.$$

Theorem 6.1 (Non-overlapping energy disposition)**.** *Under Equation* (6)*,*

$$P = F_p + X_p + L + F_c + X_c.$$

Thus the right side is a disjoint disposition of primary supply. By contrast, the gross stage sum

$$G = P + O + (F_p + F_c)$$

exceeds P whenever $O + F_p + F_c > 0$.

Proof. Substitute $C = O + L$ into $P = C + F_p + X_p$, then substitute $O = F_c + X_c$. This yields Equation (6.1). Subtracting P from Equation (6.1) gives $G - P = O + F_p + F_c$, which is positive under the stated condition. □

PROVED. This is an accounting identity, not an efficiency estimate. It formalizes claim JS-C014 in the program register.

6.1 Synthetic bridge

The pilot uses:

$$P = 1000, \quad C = 800, \quad F_p = 200, \quad X_p = 0,$$
$$O = 300, \quad L = 500, \quad F_c = 300, \quad X_c = 0.$$

The balanced disposition is

$$200 + 0 + 500 + 300 + 0 = 1000.$$

The invalid gross stage sum is

$$1000 + 300 + (200 + 300) = 1800.$$

The extra 800 J is not a measurement discovery. It is repeated energy created by adding stages.

6.2 Choice of denominator

Primary supply, final use, and useful work can each be valid denominators for a different question:

- primary supply supports resource-extraction and whole-conversion-chain analysis;
- final energy supports end-user energy-intensity analysis;
- useful physical work supports end-use conversion analysis; and
- operational facility electricity supports a bounded computing-service measurement.

The account publishes each as a typed denominator. It does not add them. A comparison must use the same energy convention or provide a conversion bridge.

7 Operational and embodied scopes

7.1 Two views, one upstream flow

Operational energy records electricity and fuels used while the measured activity runs. Embodied energy allocates energy from equipment, building, and material supply chains over a declared lifecycle. The views answer different questions.

At economy scale, current-period equipment manufacturing energy is already operational use in the manufacturing sector's physical account. Allocating that energy to a data center as embodied energy and then adding it to the economy-wide operational total counts the same flow twice. A lifecycle product account may legitimately combine operational energy with an allocated embodied increment, but it must identify the upstream components and remove any overlap with the comparison total.

7.2 Component ledger

Let each physical component have a stable identifier c, a scope label $\sigma(c)$, a period, an origin, and joules E_c. The permitted scope labels are operational and embodied increment. For selected scopes A,

$$E(A) = \sum_{c:\sigma(c)\in A} E_c.$$

The account requires component identifiers to be unique inside a reported total. If a component appears twice, reconciliation fails.

Proposition 7.1 (Duplicate-component control)**.** *If component identifiers are unique and each identifier represents one non-overlapping physical flow under the declared temporal boundary, Equation* (7.2) *contains no repeated component.*

Proof. The sum indexes a set of identifiers. Uniqueness permits each identifier at most once. The additional premise that identifiers correspond to non-overlapping physical flows prevents two different identifiers from naming the same underlying flow. □

PROVED under the identifier premise. Establishing that field records satisfy the premise is CONDITIONAL.

7.3 Required embodied-energy metadata

An embodied increment should state:

- equipment or structure covered;
- lifecycle inventory database and version;
- geographic and technology assumptions;
- manufacturing and transport stages;
- expected service life;
- utilization and allocation rule;
- treatment of replacement and end of life;
- relationship to current territorial operational flows; and
- uncertainty range.

Without these fields, operational and lifecycle values remain incomparable even when both are expressed in joules.

8 The production reconciliation

8.1 Synthetic production account

With taxes on products of 20 and subsidies on products of 5,

$$\text{GDP} = 360 + 20 - 5 = 375.$$

This number remains 375 after the outcome panels are attached.

Sector	Gross output	Intermediate use	GVA
Manufacturing	500	300	200
AI services	300	180	120
Settlement services	100	60	40
Total	900	540	360

Table 2: Synthetic production-account rows in a declared currency and price basis. Each row satisfies output minus intermediate consumption equals GVA.

8.2 The monetary non-addition rule

Suppose the compiler has:

- an estimate $A = 60$ of AI-enabled output already recorded within the 300 units of AI-service gross output; and
- gross positions $B = 1000$ protected by a settlement system during a stated window.

Neither is an addition to GDP. A is a decomposition or attribution of recorded output. Adding it yields the same production twice. B is an exposure stock or gross transaction position, not value added produced during the period.

Theorem 8.1 (GDP preservation under memorandum outcomes)**.** *Let* GDP *be reconciled by Equation* (4.1)*. Let A be a monetary memorandum amount already included in some Y_j or* GVA_j*, and let B be a monetary stock, exposure, or gross transaction amount rather than a production flow. Then*

$$\mathrm{GDP}^{\text{satellite}} = \mathrm{GDP},$$

and neither $\mathrm{GDP} + A$*,* $\mathrm{GDP} + B$*, nor* $\mathrm{GDP} + A + B$ *is the SNA production measure.*

Proof. Because A is already included in an output or GVA term used in Equation (4.1), adding it repeats recorded production. Because B is not a value-added flow under the production boundary, it is not a term in the production identity. Attaching either as a memorandum does not change the terms of Equation (4.1). □

PROVED. Classification of a real empirical amount as already-in-GVA, stock exposure, or boundary extension requires source-specific evidence.

8.3 When an extended account is appropriate

An analyst may estimate consumer surplus, unpaid work, free digital services, or externalities beyond the integrated production boundary. The 2025 SNA describes these as possible extended-account concepts and requires internal consistency and distinct terminology [3]. Such estimates can be valuable. They should appear in a separate extended panel with a bridge back to the integrated account.

The present pilot does not impute them. It avoids presenting a broad welfare aggregate without a defensible valuation and counterfactual.

9 AI production, adoption, and outcome tables

9.1 Monetary production already exists

AI software may be produced for sale, licensed as a service, developed on own account, embedded in equipment, or used internally. The 2025 SNA gives AI software an "of which" position within computer software and distinguishes software, data, and equipment elements [3]. Monetary supply-use tables should identify these activities as far as source data and classifications permit.

The satellite adds service and outcome quantities. It does not add the market value of those services again.

9.2 Balanced outcome chain

The synthetic AI chain is:

AI outcome-chain item	Quality-adjusted tasks
Produced	1,000
Adopted by the downstream process	900
Realized under the declared outcome rule	800
Not adopted	100
Adopted but not realized	100

Table 3: Synthetic balanced AI outcome chain. Produced equals adopted plus not adopted. Adopted equals realized plus adopted but not realized.

The adoption rate is $900/1000 = 0.9$. The realization rate conditional on adoption is $800/900$. Neither rate is a value weight. Both depend on the population of eligible tasks and the operational definition of realization.

9.3 Required AI indicator fields

Each AI outcome row should name:

- model and software version;
- hardware, precision, batch, and latency policy;
- task family and sampling frame;
- difficulty and quality scoring rule;
- successful and failed requests;
- unit of produced service;
- eligible adopter population;
- adoption event;
- realized-outcome rule and observation window;
- counterfactual or baseline where causal language is used;

- operational energy boundary; and
- uncertainty and missing-data treatment.

Tokens are not an outcome unit. They may help explain computational workload, but tokenizer changes can alter counts without altering service. Quality- adjusted tasks or domain-specific verified outcomes are preferable when the measurement contract is published [23, 24].

9.4 Aggregation

Additive task quantities can aggregate across establishments only when their task and quality definitions match. Adoption rates should be recomputed from summed numerators and denominators, not averaged without weights. Realized health, education, legal, and operational outcomes require domain-specific units and should not be summed merely because each is called useful.

The reference code groups outcome chains by indicator name and unit. It sums produced, adopted, and realized counts, then derives residuals and rates. It does not aggregate mismatched units.

10 Proof-of-work assurance memorandum

10.1 Scenario fields

A proof-of-work memorandum row contains:

- network and consensus-rule identifier;
- observation and settlement window;
- confirmation depth;
- adversarial-share assumption;
- hardware-access and propagation assumptions;
- named reorganization or settlement-risk model;
- operational energy scope;
- protected exposure;
- modeled expected loss;
- modeled avoided loss relative to a stated baseline; and
- cost and incentive assumptions where attack margin is reported.

The row is not "trust." It is a set of scenario-conditioned quantities.

10.2 Synthetic assurance row

The fixture uses:

$$E = 50 \text{ J}, \quad z = 6, \quad q = 0.1,$$

$$R_0 = 0.01, \quad R_1 = 0.002, \quad L_P = 1000 \text{ USD}.$$

Here R_0 is baseline reorganization probability, R_1 is scenario probability, and L_P is gross protected exposure. The modeled expected loss in the scenario is

$$R_1 L_P = 2 \text{ USD}.$$

Modeled avoided loss relative to the baseline is

$$(R_0 - R_1) L_P = 8 \text{ USD}.$$

The conditional avoided-loss intensity is

$$\frac{8}{50} = 0.16 \text{ USD/J}.$$

These are synthetic model outputs. The 1,000 USD exposure is not an output and is not added to GDP. The 8 USD avoided-loss memorandum is not added either. It describes a modeled counterfactual outcome and may overlap with recorded insurance, risk-management, or payment-service production.

10.3 Aggregation limits

Assurance rows with different networks, confirmation depths, adversarial shares, events, or windows are not directly additive. Expected losses may be summed only for non-overlapping exposures under compatible probability models. Gross positions that turn over repeatedly within a period can be much larger than the stock at risk at any instant. Reporting both avoids confusing payment volume with protected loss.

Network energy is shared. Dividing annual proof-of-work electricity by annual transactions is an allocation average, not marginal transaction energy. National accounts should report the network and time boundary, not imply that each transaction caused a fixed share of global energy.

11 The vector satellite-account architecture

11.1 Panel A: monetary production

Panel A contains:

- gross output by standard and thematic industry;
- intermediate consumption;
- GVA;
- compensation, taxes, depreciation, depletion, and operating surplus where source accounts support the breakdown;
- taxes less subsidies on products; and
- reconciliation to published GDP.

AI and settlement categories are breakouts or alternative aggregations. Their sum must reconcile to the source industry totals.

11.2 Panel B: physical energy

Panel B contains:

- product-level physical supply and use in joules;
- primary natural inputs and energy products where available;
- conversion inputs, outputs, and losses;
- final use by industry and households;
- imports, exports, and inventory changes;
- operational scope components;
- separately labeled embodied increments; and
- bridge residuals.

The product table and stage bridge are both necessary. The product table balances institutional flows. The bridge prevents an analyst from treating gross stages as independent energy.

11.3 Panel C: AI service and outcomes

Panel C contains produced, adopted, realized, and residual quantities for each typed AI indicator. It may include quality distributions, latency, reliability, human-review rates, and downstream acceptance. Monetary AI production remains in Panel A.

11.4 Panel D: settlement assurance

Panel D carries scenario-indexed proof-of-work energy, risk, exposure, expected loss, avoided loss, and attack economics. Monetary settlement-service production remains in Panel A. Gross protected assets remain exposure memoranda.

11.5 Panel E: energy-normalized vectors

Panel E forms ratios only after matching numerator and denominator boundaries. Example rows include:

- quality-adjusted adopted AI tasks per facility-operational joule;
- realized verified outcomes per lifecycle joule;
- modeled avoided settlement loss per network-operational joule;
- GVA per final-energy joule by sector; and
- GVA per primary-energy joule for economy-wide resource analysis.

These ratios answer different questions. They are not summed.

11.6 Panel F: optional scalar views

Panel F contains normalized indicators, declared weights, scores, and sensitivity. It is optional because the vector account is complete without a scalar. If published, the panel must be labeled analytical rather than an SNA aggregate.

12 Weighting and sensitivity

12.1 Normalization

For indicator x_k with declared bounds $a_k < b_k$, a min-max normalization is

$$z_k = \frac{x_k - a_k}{b_k - a_k}$$

when higher values are preferred. When lower values are preferred,

$$z_k = 1 - \frac{x_k - a_k}{b_k - a_k}.$$

Bounds can be policy thresholds, scientific limits, or empirical reference ranges. The choice changes the index and must be published. Out-of-range values require a declared clipping or rebasing rule.

12.2 Weights are normative

Weights in Equation (4.5) express importance and permitted compensation. A high AI outcome can offset weak assurance in a linear index only because the aggregation rule allows it. Equal weights do not remove this judgment. They state that normalized components receive equal marginal weight.

Proposition 12.1 (Weight-dependent rank reversal)**.** *Consider reporting units A and B with two normalized outcomes*

$$z(A) = (0.9, 0.2), \qquad z(B) = (0.3, 0.8).$$

Under weights $w^{(1)} = (0.8, 0.2)$, A ranks above B. Under $w^{(2)} = (0.2, 0.8)$, B ranks above A.

Proof. For $w^{(1)}$,

$$I_{w^{(1)}}(A) = 0.8(0.9) + 0.2(0.2) = 0.76,$$
$$I_{w^{(1)}}(B) = 0.8(0.3) + 0.2(0.8) = 0.40.$$

For $w^{(2)}$,

$$I_{w^{(2)}}(A) = 0.2(0.9) + 0.8(0.2) = 0.34,$$
$$I_{w^{(2)}}(B) = 0.2(0.3) + 0.8(0.8) = 0.70.$$

The order reverses. $\square$

PROVED. The example shows why a scalar is not a natural country ranking.

12.3 Sensitivity report

At minimum, a scalar release should report:

- component values and units before normalization;
- normalization formulas and bounds;
- direction of preference;
- missing-data and imputation rules;
- every published weight vector;
- aggregation method and compensability;
- scores and ranks under alternative plausible choices; and
- which comparisons change sign or order.

The reference code produces scores and deterministic rankings for each weight vector. The rank reversal remains visible. It does not choose a preferred vector.

13 A complete synthetic pilot

13.1 Reconciliation dashboard

Check	Result	Unit	Status
Primary-fuel supply minus use	0	J	balanced
Electricity supply minus use	0	J	balanced
Primary allocation	$1000 = 800 + 200$	J	balanced
Conversion balance	$800 = 300 + 500$	J	balanced
Converted-output allocation	$300 = 300 + 0$	J	balanced
Non-overlapping disposition	1000	J	equals primary
Invalid gross stage sum	1800	J	rejected as total
Total GVA	360	currency	reconciled
GDP	375	currency	reconciled
AI monetary memorandum	60	currency	not added
Protected-position memorandum	1000	USD	not added

Table 4: Synthetic reconciliation dashboard. All quantities are invented for testing.

13.2 Outcome dashboard

Indicator	Quantity	Unit	Interpretation
AI service produced	1,000	quality-adjusted tasks	typed output
AI service adopted	900	quality-adjusted tasks	downstream use
AI outcome realized	800	quality-adjusted tasks	declared rule
AI adoption rate	0.90	ratio	adopted over produced
AI realization rate	0.889	ratio	realized over adopted
PoW protected exposure	1,000	USD	stock or position memo
PoW expected loss	2	USD	scenario memo
PoW modeled avoided loss	8	USD	baseline contrast
PoW avoided loss intensity	0.16	USD/J	conditional ratio

Table 5: Synthetic vector outcome dashboard. Rows are not additive.

13.3 Why the pilot is bounded

The pilot demonstrates identities and failure controls. It does not estimate a country. It omits imports, inventory changes, institutional sectors, price and volume decomposition, capital accounts, labor, emissions, detailed energy carriers, uncertainty distributions, and sampling. A production pilot would start with a narrow set of industries and publish coverage gaps.

14 Executable reference artifact

The companion module `src/joule_standard/national_accounts.py` uses the Python standard library and exact decimal arithmetic. Binary floating-point inputs are rejected at the accounting boundary. The main objects are:

- `PhysicalEnergySupplyUse` and product balances;
- `EnergyStageBridge`;
- `EnergyScopeLedger`;
- `SectorProduction` and `ProductionAccount`;
- `OutcomeChain`;
- `ProofOfWorkAssuranceMemo`;
- `NationalSatelliteAccount`;
- `energy_normalized_vector`;
- `BoundedIndicator` and `WeightVector`; and
- `weight_sensitivity`.

14.1 Tested properties

Seventeen dedicated tests check:

1. physical supply equals use by product;
2. product discrepancies identify the unbalanced carrier;
3. the energy-stage bridge reconciles;
4. an inconsistent conversion balance is rejected;
5. operational and embodied ledgers reject duplicate components;
6. sector GVA and production-side GDP reconcile;
7. produced, adopted, and realized outcome chains balance;
8. aggregation requires matching indicator and unit;
9. protected exposure differs from expected loss and does not change GDP;
10. energy normalization preserves a vector of separate outcomes;
11. composites require matching indicator names and weights summing to one;
12. duplicate composite-indicator names are rejected;
13. normalized values must remain inside declared bounds;
14. an avoided-loss scenario cannot have risk above its baseline;
15. normative weights cannot be negative;
16. alternative weights expose a rank reversal; and
17. binary floats are rejected.

Passing tests are COMPUTATIONAL. They show that the reference fixture implements the stated identities. They do not establish empirical coverage, classification accuracy, causal attribution, or national representativeness.

14.2 Determinism and auditability

Every monetary and physical fixture is constructed from integer, string, or decimal inputs. Product balances are sorted by product name. Composite scores are calculated in sorted indicator order. Ties in rankings resolve by reporting unit name. These choices make test output repeatable.

The implementation is intentionally a reference model rather than a data warehouse. A statistical office would need confidential-data controls, classification services, revision vintages, provenance, seasonal adjustment, price and volume measures, and dissemination infrastructure.

15 Compilation protocol

15.1 Phase 1: mandate and boundary

The compiler should publish:

- the policy question;
- relationship to the integrated SNA account;
- SNA edition used by source data;

- SEEA version and energy classifications;
- residence and geographic boundary;
- reference period and revision policy;
- operational, final, primary, and lifecycle energy conventions; and
- the list of memorandum outcomes.

15.2 Phase 2: monetary disaggregation

Identify AI and settlement activities within source industries and products. Disaggregate output, intermediate consumption, and GVA while preserving source totals. Publish allocation methods and residuals. Reconcile the thematic table to the standard supply-use and production accounts.

No monetary outcome is added at this phase. If an AI activity is internal ancillary production, the compiler follows SNA treatment and may publish a thematic breakout without altering the integrated total.

15.3 Phase 3: physical energy account

Compile product-level supply and use in original units, convert to joules using published calorific factors, and balance each carrier. Build the stage bridge. Report primary supply, conversion, losses, final use, and other dispositions as linked views. Do not publish their gross sum as energy input.

Metered facility data should be preferred to equipment nameplate power. Allocation from facilities to workloads requires time-resolved evidence or a declared model.

15.4 Phase 4: scope ledger

Create stable identifiers for operational and embodied components. State whether embodied energy is an upstream current-period flow, a historical flow, or an amortized lifecycle allocation. Reconcile overlaps before forming a lifecycle total.

15.5 Phase 5: outcome data

Define each indicator before collecting it. Build produced, adopted, realized, and residual fields. Preserve numerators and denominators for rates. Record missing data and selection. If "realized" implies causality, specify the counterfactual and identification design.

15.6 Phase 6: proof-of-work scenarios

Define the network, rules, event, confirmation depth, adversarial share, hardware access, energy, costs, benefit, propagation, and time window. Keep gross exposure, modeled probability, expected loss, and avoided loss separate.

15.7 Phase 7: ratios and sensitivity

Form energy-normalized vectors only when numerator and denominator boundaries match. If a composite is requested, publish normalization, weights, alternative weight vectors, and rank sensitivity. Retain the underlying vector in every release.

15.8 Phase 8: validation and dissemination

Run:

- product-level supply-use checks;
- stage-bridge identities;
- duplicate-component checks;
- GVA and GDP reconciliation;
- outcome-chain balances;
- stock-flow and unit checks;
- memorandum non-addition checks;
- classification residual checks; and
- weight-sensitivity analysis.

Release tables, methods, revisions, and machine-readable metadata together.

16 Failure modes

16.1 Adding energy stages

Primary fuel enters a power plant, part becomes electricity, and electricity reaches a user. Adding fuel energy, electricity output, and final electricity as independent input counts the conversion chain more than once. The stage bridge, not a label change, fixes the problem.

16.2 Adding AI value twice

AI software sales, cloud services, own-account software, and AI-enabled business output can already affect output, intermediate consumption, capital formation, and GVA. An outcome estimate may explain or decompose those entries. Adding the estimate to GDP repeats production unless it represents a distinct boundary extension with a complete extended account.

16.3 Calling protected assets output

A settlement system can protect positions much larger than its service production. Gross protected assets are an exposure base. Avoided expected loss is a counterfactual model result. Neither equals value added.

16.4 Mixing operational and lifecycle energy

Facility electricity excludes manufacturing and construction. A lifecycle estimate may include them. Comparing or adding the two without a component bridge changes the boundary while preserving the displayed unit.

16.5 Averaging rates

The unweighted average of establishment adoption rates generally differs from the national adoption rate. The correct aggregate for a common definition is the sum of adopted units divided by the sum of eligible or produced units.

16.6 Using equal weights as neutrality

Equal weights make normalized components equally important in the linear index. They are not absence of judgment. They can also assign greater total weight to a dimension represented by more indicators.

16.7 Interpreting intensity as causality

A sector with high GVA per joule may have a different product mix, capital stock, labor force, geography, or energy boundary. The ratio is descriptive. A claim that additional energy caused additional value needs a causal design.

17 Uncertainty, revisions, and comparability

17.1 Source uncertainty

Energy data may combine meters, surveys, engineering estimates, calorific factors, and modelled allocations. AI outcomes may come from vendor logs, enterprise surveys, benchmarks, or administrative records. Proof-of-work assumptions may be scenario ranges rather than observations. Each source needs coverage, sampling, imputation, and uncertainty metadata.

17.2 Revision vintages

GDP, supply-use tables, energy accounts, and digital indicators are revised on different schedules. The satellite should carry a vintage for every panel and publish a revision triangle or change log. Combining a revised GDP denominator with an old energy numerator can create a spurious change in intensity.

17.3 Cross-country comparability

Countries differ in source data, SNA implementation stage, energy residence adjustments, industrial structure, AI coverage, and disclosure rules. A common template improves comparability but does not guarantee it. The release should publish a comparability matrix and avoid rankings where coverage is not aligned.

17.4 Weight uncertainty

Weight sensitivity is not a substitute for source uncertainty. Both matter. The account should vary data, normalization, inclusion, weights, and aggregation where plausible, consistent with OECD/JRC guidance [11].

18 Policy uses and limits

18.1 Productivity diagnosis

The vector account can distinguish:

- a monetary change in GVA;
- a physical change in final or primary energy;
- a hardware-efficiency change;
- a change in quality-adjusted AI service;
- an adoption bottleneck;
- a realized-outcome bottleneck; and
- a settlement-risk change under fixed scenario assumptions.

That decomposition is more informative than one aggregate value-per-joule number.

18.2 Industrial and digital policy

A government can use the thematic production table to see where AI software, cloud, equipment, and internal use appear. Outcome chains can reveal whether service production reaches downstream processes. Energy panels can show operational and upstream requirements. Policy evaluation still needs counterfactual evidence.

18.3 Energy planning

The account can show which industries use final electricity and how conversion losses connect primary supply to end use. It cannot determine the marginal grid effect of a new data center from annual averages. Nodal timing, network constraints, and marginal generation require a separate power-system model.

18.4 Settlement policy

Scenario memoranda can help regulators and institutions distinguish payment volume, protected exposure, expected loss, and energy. The account does not certify a network or convert one model into a legal-finality rule.

19 Limitations

Limitation 19.1 (Synthetic data). Every numerical table is invented. No result describes a country, industry, AI system, or proof-of-work network.

Limitation 19.2 (No complete SNA sequence). The pilot contains a production account, not the full sequence of institutional sector accounts, accumulation accounts, balance sheets, input-output tables, or rest-of-world account.

Limitation 19.3 (Simplified physical table). The energy table omits natural-input detail, imports, exports, inventories, residuals, multiple fuels, and residence-territory adjustments.

Limitation 19.4 (Outcome definitions). Produced, adopted, and realized quantities are only as credible as their operational definitions and evidence. Arithmetic balance does not establish causal value.

Limitation 19.5 (Proof-of-work model). The memorandum uses supplied scenario probabilities. It does not estimate live network risk, censorship resistance, liveness, or total security.

Limitation 19.6 (Embodied energy). Unique component identifiers prevent literal duplicate rows but cannot prove that two differently named inventory items do not overlap. Provenance and physical modelling remain necessary.

Limitation 19.7 (Composite index). The reference composite uses min-max normalization and linear aggregation. It does not establish that compensation across outcomes is normatively acceptable.

20 Research agenda

20.1 Classification concordances

Develop concordances among standard industries, digital supply-use categories, AI product types, computing facilities, energy carriers, and settlement activities. Publish straddle rules for mixed establishments.

20.2 Outcome measurement standards

Create indicator dictionaries for quality-adjusted machine service, adoption, human review, downstream acceptance, and realized outcomes. Define sampling, benchmark drift, and privacy-preserving aggregation.

20.3 Embodied-energy bridges

Link PEFA with input-output and lifecycle inventories while preserving period, residence, and component identity. Quantify overlap rather than merely warning about it.

20.4 Assurance exposure accounts

Develop scenario classes for reorganization, censorship, inclusion delay, and operational failure. Separate gross positions, peak exposure, expected loss, insurance, and recourse.

20.5 Revision and uncertainty propagation

Propagate source revisions and probability distributions through ratios and composites. Report confidence intervals and sensitivity without implying that statistical uncertainty captures normative uncertainty.

20.6 Bounded national pilots

Begin with one or two industries and a small outcome set. Require full reconciliation before expanding. A narrow balanced account is preferable to a large dashboard with hidden overlap.

21 Conclusion

The 2025 SNA now provides the current international framework for national accounts, while SEEA 2012 and SEEA-Energy provide the operative environmental and energy accounting structure. Together they support a disciplined answer to the value-per-joule national-accounting question.

The answer is not to enlarge GDP with intelligence or trust. AI-related output can already be recorded in production. Protected asset positions are not value added. Primary energy, converted energy, and final energy are connected stages, not independent inputs. Operational and embodied scopes can describe the same upstream flow from different perspectives.

A vector satellite account preserves these distinctions. Monetary production reconciles to GDP. Physical energy balances by product and through a conversion bridge. AI outcomes balance from production through adoption to realization. Proof-of-work assurance remains tied to a named scenario. Energy-normalized outcomes retain their units. Optional scalar views publish their normative weights and rank sensitivity.

This architecture supplements GDP with useful physical and outcome information while leaving the production boundary intact. Its discipline lies less in producing one new number than in preventing several familiar numbers from being added when they should remain separate.

A Symbol and unit register

Symbol	Meaning	Unit
Y_j	gross output of sector j	currency/period
IC_j	intermediate consumption of sector j	currency/period
GVA_j	$Y_j - IC_j$	currency/period
T_p, S_p	taxes and subsidies on products	currency/period
GDP	integrated production-side GDP	currency/period
S_{ra}	supply of energy product r by actor a	J/period
U_{rb}	use of energy product r by actor b	J/period
P	primary energy-product supply	J/period
C	primary energy sent to conversion	J/period
F_p	direct final use of primary energy	J/period
X_p	other primary-energy disposition	J/period
O	converted-energy output	J/period
L	conversion loss	J/period
F_c	final use of converted energy	J/period
X_c	other converted-energy disposition	J/period
Q_k^{produced}	produced typed outcome k	indicator-specific
Q_k^{adopted}	adopted typed outcome k	indicator-specific
Q_k^{realized}	realized typed outcome k	indicator-specific
R_0, R_1	baseline and scenario risk	probability
L_P	protected exposure	currency stock
z_k	normalized indicator	$[0, 1]$
w_k	published normative weight	nonnegative share
I_w	declared composite view	index

Table 6: Units are part of the account and must accompany every release.

B Pilot identity worksheet

B.1 Physical product checks

$$\begin{aligned}
\text{Primary-fuel supply} &= 1000, \\
\text{Primary-fuel use} &= 800 + 200 = 1000, \\
\text{Electricity supply} &= 300, \\
\text{Electricity use} &= 120 + 100 + 50 + 30 = 300.
\end{aligned}$$

B.2 Stage checks

$$\begin{aligned}
P &= C + F_p + X_p = 800 + 200 + 0, \\
C &= O + L = 300 + 500, \\
O &= F_c + X_c = 300 + 0, \\
P &= F_p + X_p + L + F_c + X_c = 1000.
\end{aligned}$$

B.3 Production checks

$$\begin{aligned}
\mathrm{GVA}_{\mathrm{manufacturing}} &= 500 - 300 = 200, \\
\mathrm{GVA}_{\mathrm{AI}} &= 300 - 180 = 120, \\
\mathrm{GVA}_{\mathrm{settlement}} &= 100 - 60 = 40, \\
\textstyle\sum_j \mathrm{GVA}_j &= 360, \\
\mathrm{GDP} &= 360 + 20 - 5 = 375.
\end{aligned}$$

B.4 Outcome checks

$$1000 = 900 + 100, \qquad 900 = 800 + 100.$$

$$R_1 L_P = 0.002(1000) = 2,$$

$$(R_0 - R_1) L_P = (0.01 - 0.002)(1000) = 8.$$

C Minimum release schema

Panel	Required keys	Required checks
Monetary production	period, industry, product, output, intermediate use, GVA, price basis	row GVA and GDP reconciliation
Physical energy	carrier, supplier, user, joules, residence, period	product supply equals use
Stage bridge	primary, conversion input, output, losses, final and other use	three bridge identities
Scope ledger	component ID, operational or embodied, origin, period, joules	unique IDs and overlap review
AI outcomes	indicator, model, task frame, produced, adopted, realized, unit	two outcome-chain balances
PoW assurance	scenario, network, window, confirmations, risk, exposure, energy	unit, scenario, and stock-flow checks
Composite	raw indicators, bounds, orientation, weights, method	weights sum to one and sensitivity

Table 7: Minimum machine-readable release schema for the proposed account.

D Artifact-to-claim map

Claim	Status	Evidence
Energy stages cannot be added independently	PROVED	theorem 6.1 and bridge tests
Satellite outcomes do not alter GDP	PROVED	theorem 8.1 and production tests
Physical product tables balance	COMPUTATIONAL	`test_national_accounts.py`
Outcome chains reconcile	COMPUTATIONAL	outcome-chain tests
Protected exposure is not expected loss	COMPUTATIONAL	assurance memo test
Weights can reverse a ranking	PROVED and COMPUTATIONAL	proposition 12.1 and sensitivity test
National source data fit the proposed schema	OPEN	requires a bounded pilot
Outcome attribution is causal	OPEN	requires indicator-specific identification

Table 8: The software supports identities and fixtures, not national estimation claims.

E Standards-status note

This manuscript was researched on 23 July 2026. On that date:

- the 2025 SNA was the adopted international statistical standard;
- UNSD distributed its searchable pre-edit version;
- implementation work and compilation guidance were ongoing;
- the SEEA 2012 Central Framework remained the adopted standard;
- an update of the SEEA Central Framework was in consultation; and
- the 2019 SEEA-Energy publication remained the energy-accounting subsystem used here.

Future users should recheck status before describing an edition as current.

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